

# Bounds on Burning Number for Butterfly-Type Networks

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## Abstract

Interconnection networks such as the Butterfly, Benes, and Wrapped Butterfly networks are fundamental structures in parallel and distributed systems due to their regular topology, hierarchical layering, and efficient routing properties. These networks exhibit exponential growth in the number of vertices with respect to the number of levels, which makes the exact determination of their burning number particularly challenging. The burning number of a graph is a graph parameter that models the spread of information (or contagion) over discrete time steps; it is defined as the minimum number of steps required to ensure that all vertices of the graph are *burned*, where at each step a new vertex can be ignited and the fire simultaneously spreads to adjacent vertices. In this paper, we present a graph-theoretic study of these networks with an emphasis on the burning process as a model for information propagation. Each network is formally defined, and its structural properties are examined. We establish lower and upper bounds for the burning number of Butterfly, Benes, and Wrapped Butterfly networks in terms of key parameters such as the number of levels and the graph radius. These bounds capture the influence of the networks' layered architecture on the spread of the burning process and provide insight into burning dynamics on large-scale structured networks.

**Keywords:** Graph burning; Burning number; Butterfly Network; Benes Network; Wrapped Butterfly Network.